# THE CORRELATION BETWEEN TEMPERATURE DEPENDENCE OF ELECTRICAL RESISTANCE, RESISTIVITY, AND THERMAL CONDUCTIVITY OF METALS

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A correspondence between certain transfer characteristic parameters is exposed by generalization of experimental data. The results may be used to analyze reliability of experimental data and to extrapolate the temperature dependence of thermal or electrical conductivity.

According to elementary theory [1] the temperature dependence of resistivity in metals at high temperatures (significantly exceeding the characteristic temperature) is linear. For the overwhelming majority of metals this principle is fulfilled as a first approximation; in fact there occur deviations from linearity in both a positive and negative direction (in the terminology of [2, 3] this factor is one of the characteristics by which all metals may be divided into "plus" and "minus" groups). These deviations from linear temperature dependence can be ascribed to peculiarities in the electron structure of real metals [2-7].

It is natural to expect that factors producing one or the other character of temperature dependence of resistance will exert an effect on certain other transfer parameters, creating a correlation between their values. Using analysis of experimental data for eleven transition metals, [6] established that the relative change with temperature of the quantity $\rho/T$ correlates with the absolute values of $\rho$. A generalization of experimental data for both transition and nontransition metals performed by the present author indicated the presence of a general tendency for the value of the electrical resistivity at any temperature above characteristic to contain certain information on the temperature dependence of the resistivity.

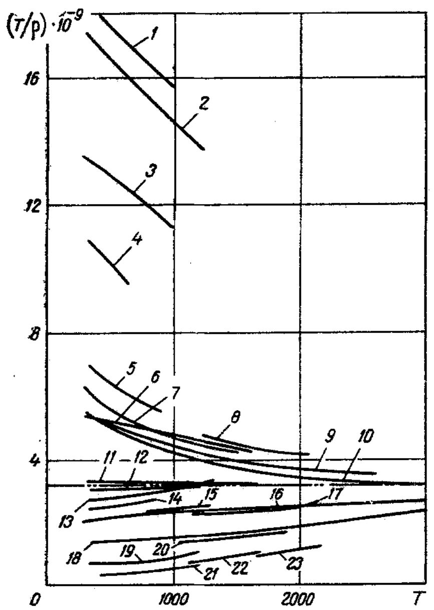


Fig. 1. $(T/\rho)\cdot 10^{-9}$ (°K · $\Omega^{-1}m^{-1}$) of metals temperature (°K): 1) silver [9]; 2) copper [8]; 3) gold [9]; 4) aluminum [10]; 5, 6, 7) magnesium, iridium, rhodium [8]; 8) rhodium [11]; 9, 10) molybdenum, tungsten [12]; 11, 12) osmium, platinum [12]; 13) palladium [8]; 14) palladium [13]; 15) nickel above Curie point [8]; 16) tantalum [14]; 17) niobium [15]; 18) rhenium [16]; 19) zirconium [8]; 20) vanadium [17]; 21) gadolinium [18]; 22) titanium [19]; 23) hafnium [20].

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In further development it will be convenient to use an approximation in the form of the Worting formula

$$\rho = AT^n \tag{1}$$

(in the general case the entire temperature range may be divided into subranges with their own constant values of A and n). Linear dependence of $\rho$ on T obviously corresponds to the condition $n = 1$. For $n < 1$ there is negative deviation from this dependence; for $n > 1$, positive. Further, for $n < 1$, $T/\rho$ as a function of temperature is increasing; and for $n > 1$ it is decreasing. Moreover, the first case corresponds to a negative second derivative of resistance with respect to temperature, while the second corresponds to a positive second derivative.

Figure 1 presents $T/\rho$ as a function of T for 21 metals, including both transition and nontransition metals. Data were taken from reference [8] and original studies, most of recent origin [9-20], including studies of the present author [12, 14, 16]. It is evident from the figure that with respect to dependence on absolute value of the ratio $T/\rho$ (or $\rho$ value at identical temperature) all the curves may be divided into two groups; those with falling and rising temperature dependence of $T/\rho$. The limiting value of the ratio $T/\rho$ is approximately $3.2 \cdot 10^9\,{}^\circ\mathrm{K} \cdot \Omega^{-1} \cdot \mathrm{m}^{-1}$ (dashed line). A natural exception to the general principle is seen in data for metals with specific magnetic properties, not shown in the figure: nickel below the Curie point, iron, cobalt, and chromium.

Thus, for the majority of metals the value of $T/\rho$ even at a single temperature permits determination of the character of the deviation of the temperature dependence of resistance from linearity.

By extrapolation of the curves of Fig. 1 to the right one can find a certain range of values corresponding to the intersection of the individual curves with the limit line. The abscissa of this range is $T_i \approx 4000\,{}^\circ\mathrm{K}$, with ordinate $B \approx 3.2 \cdot 10^9\,{}^\circ\mathrm{K} \cdot \Omega^{-1} \cdot \mathrm{m}^{-1}$. With a linear extrapolation rule and use of the constants $T_i$, B, one can obtain an equation roughly describing the temperature dependence of resistivity for each metal

$$\rho = \frac{T}{B + \left(\frac{T_1}{\rho_1} - B\right)\frac{T_\text{п} - T}{T_\text{п} - T_1}}. \tag{2}$$

The experimental value of $\rho$ at some temperature $T_1$ appears in the equation.

Despite the requirements of classical electron conductivity theory [5], the coefficient of thermal conductivity of the majority of metals above the characteristic temperature is a function of temperature. In many cases it is useful to know the sign of the temperature dependence $\lambda$, for example, in extrapolating temperature data. At the present time it is not possible to predict beforehand the behavior of this dependence from the physicochemical constants of the metal. However, commencing from the close relationship between heat and charge transfer in metals, it may be expected that there would be a correspondence in the functions $\lambda$ and $T/\rho$ as functions of T. From the equation $\lambda(T)/[T/\rho(T)] = \text{const}$ expressing the Wideman-Frantz law, one can obtain a relationship defining the sign of the temperature dependence of the coefficient of thermal conductivity:

$$d\lambda/dT \sim d(T/\rho)/dT. \tag{3}$$

With consideration of the facts brought forth earlier, it may be added that

$$d\lambda/dT \sim -d^2\rho/dT^2, \tag{4}$$

i.e., thermal conductivity decreases with temperature if there exists a "concave" dependence of $\rho$ on T, and increases if there is a "convex" dependence.

The existence of correlation between the temperature dependence of $\lambda$ and $\rho$ adequately satisfying Eqs. (3, 4) was noted in [6, 21, 22] for a series of transition metals. Analysis of experimental data shows that these relationships are fulfilled for practically all metals which have been studied adequately.

Figure 2 shows $\lambda$ as a function of $T/\rho$ for 18 metals, both transition and nontransition. It is evident that the derivative $d\lambda/d(T/\rho)$ for all metals is greater than zero, which indicates the validity of Eq. (3). The figure does not show data for copper, silver, and gold due to the high values of $T/\rho$ and $\lambda$, although they do obey the general principles. It is interesting that for iron and nickel the change in sign of the temperature dependence of $\lambda$ upon passage through the Curie point leads to the presence of two branches in the curves for these metals in Fig. 2, each separate branch conforming to the general principle.

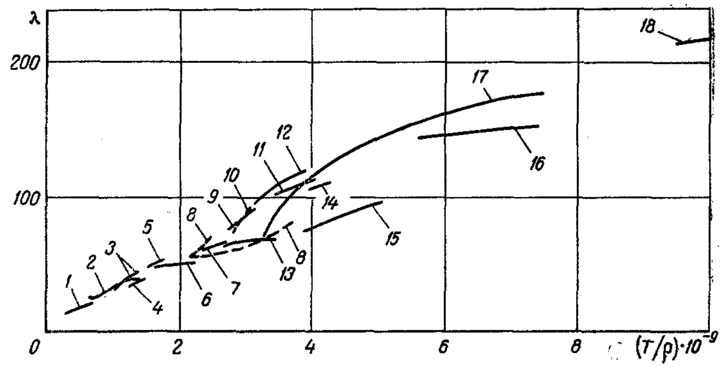


Fig. 2. Coefficient of thermal conductivity λ (W/m · °K) versus $(T/\rho) \cdot 10^{-9}$ ($°K \cdot \Omega^{-1} \cdot m^{-1}$): 1) gadolinium [18]; 2) titanium [19]; 3) iron [8]; 4) zirconium [23]; 5) vanadium [17]; 6) rhenium [16]; 7) titanium [14]; 8, 9) nickel, chromium [8]; 10) palladium [13]; 11, 12) molybdenum, tungsten [12]; 13) niobium [15]; 14) irridium [24]; 15) cobalt; 16, 17) magnesium, beryllium [8]; 18) aluminum [9].

Although Eqs. (3, 4) determine the temperature behavior of λ only to a certain probability, they may still be of value in estimating the reliability of experimentally obtained data. As an example, we may consider the thermal conductivity values of tantalum and rhenium recommended in [8] as most reliable. The decrease of λ of these metals with temperature contradicts Eqs. (3, 4). Studies of a number of authors, especially those performed recently, indicate an increase in thermal conductivity with temperature in tantalum and rhenium (cf., for example, [14, 16]). In the same fashion, among the results obtained in measurement of thermal conductivity of platinum, greater credence should be given to those in which λ increases with temperature [25, 26] than those indicating a decrease [27, 28].

A rough prediction of the temperature behavior of λ may be made without recourse to data on resistivity. From the limiting value of $T/\rho$ mentioned above it is possible to find a limiting value of λ, calculating the most probable value of the proportionality coefficient between these quantities on the basis of Fig. 2. The limiting value of the coefficient of thermal conductivity obtained in this fashion is approximately 80 W/m · °K. If the thermal conductivity of the metal exceeds this value, it may be expected that it will decrease with increase in temperature and vice versa.

## NOMENCLATURE

$\rho$, electrical resistivity; λ, coefficient of thermal conductivity; T, absolute temperature; A, n, coefficients of Worting formula; $T_i$, B, coefficients of empirical formula describing dependence of $\rho$ on temperature in metals.

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